FINALTERM EXAMINATION
(Session - 4)

Calculus & Analytical Geometry-I

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UPDATED VERSION
Exclusive thanks to Mahar Azahar (Lodhran)

(Marks: 1) - Please choose one

Let the region bounded by the curve $y = \sqrt[3]{x}$, the x-axis, and the line $x = 3$ is revolved about the y-axis to generate a solid. Which of the following equation gives the volume of a solid by cylindrical shells?

$$V = \int_0^3 x^{\frac{3}{2}}\, dx$$

▶

$$V = 2\pi \int_0^3 \sqrt{x}\, dx$$

▶

$$V = \int_0^3 2\pi x \sqrt[3]{x}\, dx$$

▶

$$V = \int_0^3 x \sqrt[3]{x}\, dx$$

▶

(Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the difference between successive terms $a_{n+1} - a_n \leq 0$ then the sequence is known as:

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

(Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} < 1$ then the sequence is known as**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

(Marks: 1) - Please choose one

If $\rho = \lim_{k \to +\infty} \sqrt[k]{u_k}$ where $\rho > 1$ then the series $\sum u_k$ with positive terms will /will be.....?

► Convergent
► Divergent
► Give no information

(Marks: 1) - Please choose one

In alternating series test, which one of the following condition must be satisfied?

► $\lim_{k \to \infty} a_k = 1$
► $a_1 > a_2 > a_3 ..... > a_k > ....$
► $a_1 \le a_2 \le a_3 ..... \le a_k \le ....$

(Marks: 1) - Please choose one

Let $y = \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \; ; \; 0 \le x \le 2$ then which of the following is the length of the curve?

$$L=\int_0^2\sqrt{\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\,dx$$

►

$$L=\int\sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\,dx$$

►

$$L=\int_0^2\sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\,dx$$

►

$$L=\int_0^2\sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]}\,dx$$

►

(Marks: 1) - Please choose one

$\frac{2}{3}$ is known as

► An even number
► Irrational Number
► A natural Number

► Rational Number

(Marks: 1) - Please choose one

For a function *f*, let $f'(x_n)=0 \quad for\ some\ n$.

Does the Newton's Method works for approximating the solution of $f(x)=0$?

► Yes
► No

(Marks: 1) - Please choose one

The Mean Value Theorem states that "Let function *f* be differentiable on (a,b) and continuous on [a, b], then there exist at least one point c in (a,b) where ……. "

▶ $f'(c) = \frac{f(b) - f(a)}{b - a}$

$f(c) = \frac{f(b) - f(a)}{b - a}$

▶

$f(c) = \frac{f(a) - f(b)}{b - a}$

▶

$f'(c) = \frac{f(a) - f(b)}{b - a}$

▶

(Marks: 1) - Please choose one

$$\frac{d}{dx}[F(x)] = f(x)$$

If there is some function *F* such that then any function of the form $F(x) + C$ is ---------- of *f*(*x*)

▶ Derivative
▶ Antiderivative
▶ Slope
▶ Maximum value

(Marks: 1) - Please choose one

If *f* and *g* are continues function on an interval [a, b] and $f(x) \geq g(x) \ \ for \ \ a \leq x \leq b$, then area is bounded by the lines parallel to:

▶ X -axis
▶ Y-axis
▶ Both X -axis and Y-axis

(Marks: 1) - Please choose one

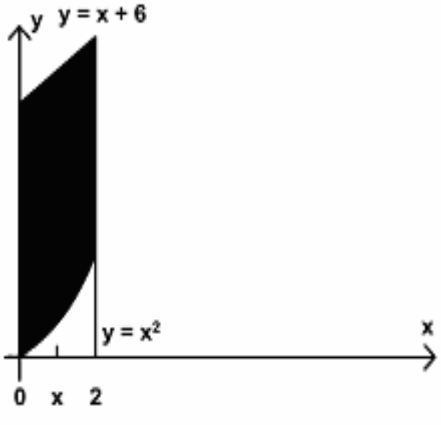
Sigma notation is represented by which of the following Greek letter?

▶ χ
▶ η

- Σ
- ψ

(Marks: 1) - Please choose one

In the following figure, the area enclosed is bounded below by **:**



- $y = x + 6$
- $y = x^2$
- $x = 2$
- $x = 0$

(Marks: 1) - Please choose one

Consider a function $h(x)$ and a constant c then

$\frac{d}{dx}\left((c)\{h(x)\}\right) =$ _____

- 0
- $\frac{d}{dx}\left(h(x)\right)$
- $\frac{d}{dx}\left(h(cx)\right)$
- $c\frac{d}{dx}\left(h(x)\right)$

(Marks: 1) - Please choose one

Let the solid generated by the region enclosed between

$y = \sqrt{x}$; $x = 1, x = 4$ and the x-axis is revolved about the y-axis. Which of the following equation gives the volumes of a solid by cylindrical shells?

$V = \int_1^4 2\pi x\sqrt{x}dx$

▶

$V = \int_1^4 2x\sqrt{x}dx$

▶

$V = \int_0^4 2x\sqrt{x}dx$

▶

$V = \int_{-4}^4 2x\sqrt{x}dx$

▶

(Marks: 1) - Please choose one

Let f is a smooth curve on the interval [a, b]. What is the arc length L of the curve f(x) defined over the interval [a, b]?

$L = \lim_{\max \Delta x \to 0} \sum_{k=1}^{n} \sqrt{1 + (f'(x^*_k))}$

▶

$L = \sum_{k=1}^{n} \sqrt{1 + (f'(x^*_k))}\Delta x_k$

▶

$L = \lim_{\max \Delta x \to 0} \sum_{k=1}^{n} \sqrt{1 + (f'(x^*_k))^2}\Delta x_k$

▶

$L = \sum_{k=1}^{n} \sqrt{1 + (f(x^*_k))}\Delta x$

▶

(Marks: 1) - Please choose one

Let $f(x)$ is a function such that as x approaches a real number a, either from left or right-hand-side, the function values increases or decreases unboundedly then

$\lim_{x \to a} f(x)$

- ► Exist
- ► Does not exist

(Marks: 1) - Please choose one

$\frac{d(\sec x)}{dx} =$

- ► $(\sec x)(\tan x)$
- ► $(\sec x)(\tan x)$
- ► $(co\sec x)(co\tan x)$
- ► $(co\sec x)(\tan x)$

(Marks: 1) - Please choose one

At what points the two curves: $y = x^2 \ and \ y = x + 6$ intersect ?

- ► $x = 0 \ and \ x = 2$
- ► $x = 0 \ and \ x = 3$
- ► $x = 2 \ and \ x = 3$
- ► $x = -2 \ and \ x = 3$

(Marks: 1) - Please choose one

If f is continuous function such that $\lim_{x \to -\infty} f(x) = +\infty \ \ and \ \lim_{x \to +\infty} f(x) = +\infty$ then f has _____ on $(-\infty, +\infty)$

- ► maximum value but no minimum
- ► minimum value but no maximum
- ► both maximum and minimum value

For a graph to be symmetric about y-axis means, for each point (x,y) on the graph,the point ---------- is also on the graph

► (x , -y)
► (-x , y)
► (-x , -y)

(Marks: 1) - Please choose one

The graph $x = y^2$ is symmetric about ---------axis

► X-axis
► Y-axis
► Origin

(Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} \geq 1$ then the sequence is known as **:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

(Marks: 1) - Please choose one

$$a_n = \left\{\frac{1}{n}\right\}_{n=1}^{\infty}$$

Which of the following option is true for the sequence ?

► Increasing
► Decreasing
► Nonincreasing
► Nondecreasing

(Marks: 1) - Please choose one

If the partial sum of a series is finite then the series will/will be**:**

► Divergent
► Convergent
► Give no information

(Marks: 1) - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + ... + ar^{k-1} + ... \ \ where\ (a \neq 0)$, $|r| < 1$ then which of the following is true for the given series?

► Converges
► Diverges
► Gives no information

(Marks: 1) - Please choose one

If $\rho = \lim_{k \to +\infty} \frac{u_{k+1}}{u_k}$ where $\rho > 1$ then the series $\sum u_k$ with positive terms will /will be.....?

► Convergent
► Divergent
► Give no information

(Marks: 1) - Please choose one

If a quantity y depends on another quantity x in such a way that each value of x determines exactly one value of y , we say that y is ..... of x

► relation
► function
► not a function
► not a relation

(Marks: 1) - Please choose one

Domain of the function y = $\frac{(x^2 - 4)}{(x - 2)}$ is

► $(-\infty, 2) U (2, +\infty)$
► $(-\infty, 2)$

▶ $(-\infty, 0)$

(Marks: 1) - Please choose one

Tan(x) is continuous every where except at points

▶ $\pm\frac{k\pi}{2}(k = 1,3,5,...)$

▶ $\pm\frac{k\pi}{2}(k = 2,4,6,...)$

▶ $\pm\frac{k\pi}{2}(k = 1,2,3,4,5,6,...)$

(Marks: 1) - Please choose one

$\lim_{x\to 0}\frac{\sin x}{x}$

= ----------

▶ -1
▶ 2
▶ 0
▶ 1

(Marks: 1) - Please choose one

How the series $1-3+5-7+9-11$ can be expressed in sigma notation?

▶ $\sum_{k=0}^{k=5}(-1)^k(2k+1)$

▶ $\sum_{k=1}^{k=5}(-1)^k(2k+1)$

▶

$$\sum_{k=1}^{k=5}(2k+1)$$

▶

$$\sum_{k=1}^{k=5}(2k+1)$$

▶

(Marks: 1) - Please choose one

What is the sum of following series?

$1^3+2^3+3^3+4^3+\_\_\_\_\_+n^3$

$$\frac{n(2n)(2n+1)}{6}$$

▶

$$\frac{(n+1)(n+2)}{2}$$

▶

$$\left[\frac{n(n+2)}{2}\right]^2$$

▶

$$\left[\frac{n(n+1)}{2}\right]^2$$

▶

(Marks: 1) - Please choose one

$\frac{5}{7}\times1^2+\frac{5}{7}\times2^2+\frac{5}{7}\times3^2+\frac{5}{7}\times4^2....+\frac{5}{7}\times n^2=$ _____

$$\frac{5n(n+1)(2n+1)}{42}$$

▶

$$\frac{5n(n+1)}{14}$$

▶

$\frac{5n^2(n+1)^2}{14}$

▶

$\frac{5(n+1)(2n+1)}{42}$

▶

(Marks: 1) - Please choose one

$\int_a^a f(x)dx =$ _____

If point a is in the domain of function f, then

▶ $f'(x)$

▶ $f(x)$

▶ 0

▶ 1

(Marks: 1) - Please choose one

If $a_1 > a_2 > a_3 > ..... > a_n > ....$, then a sequence $\{a_n\}$ is .....

▶ Increasing

▶ Nondecreasing

▶ Decreasing

▶ Nonincreasing

(Marks: 1) - Please choose one

A series of the form $\sum_{k=1}^{\infty}(-1)^n a_k$ is called _____.

▶ Alternating series

▶ Geometric series

▶ Arithmetic series

▶ Harmonic series

(Marks: 1) - Please choose one

Which of the following is the Maclaurin series for e^x ?

$$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^k}{k!}+\ldots$$

▶

$$x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^k}{k!}+\ldots$$

▶

$$1+x+\frac{x^3}{3!}+\ldots+\frac{x^k}{k!}+\ldots$$

▶

$$1-x+\frac{x^3}{3!}-\ldots-\frac{x^k}{k!}-\ldots$$

▶

(Marks: 1) - Please choose one

Which of the following is the work done *W* if an object moves in the positive direction along a coordinate line while subject to a force *F(x)* in the direction of motion over an interval [0,3]?

$$W=\int_{2}^{3}3xdx$$

▶

$$W=\int_{0}^{3}3xdx$$

▶

$$W=\int_{0}^{3}F(x)dx$$

▶

$$W=\int_{3}^{0}F(x)dx$$

▶

(Marks: 1) - Please choose one

Which of the following is the spring constant *k* if a spring whose natural length is 2*m* exerts a force of 3*N* when stretched 1*m* beyond its natural length?

▶ 3 *x*

▶ 3 *N/m*

▶ 2 *m*

► 3 *m/N*

(Marks: 2)

Find the limits of the integral indicating the area bounded by the curves $y = x^2 \; and \; y = x + 6$.

(Marks: 2)

What will be the amount of work done if an object moves 7*m* in the direction of a force of 70*N*?

(Marks: 2)

Evaluate the following integral by substitution method.

$$\int x\,(2x^2+1)^{\frac{2}{3}}\,dx$$

(Marks: 3)

Evaluate the following integral:

$$\int \frac{5-6\sin^2 x}{\sin^2 x}\,dx$$

(Marks: 3)

Find the spring constant '*k*'; if a force of 10*N* is required to stretch a spring from its natural length of 4.8*m* to a length of 6.8*m*?

(Marks: 3)

Find a definite integral indicating the area of the surface generated by revolving the curve $y = \sqrt[3]{3x} \;\; ; \;\; 0 \le y \le 4$ about the $x-$ axis. But do not evaluate the integral.

(Marks: 5)

Let $\frac{d}{dx}\left[f(x)\right] = 12x^2 - 6x + 1$. Find $f(x)$

(Marks: 5)

Determine whether the sequence $\{a_n\}$ converges or diverges; if it converges then find its limit;

$$a_n = \frac{3n^4+1}{4n^2-1}$$

where

(Marks: 5)

Use the cylindrical shell to find the volume of the solid generated when the region enclosed by the curve $y = x^3, x = 1, y = 0$ is revolved about the *y*-axis.

(Marks: 10)

Find the area of the region that is enclosed by the curves $y = x^2$ and $y = \sqrt{x}$ between $x = \frac{1}{4}$ and $x = 1$.

This paper is solved by our best knowledge. In the case of any error/correction/suggestion, please contact at gulshanvu@yahoo.com, with reference to the concerned paper's number.